# Cutting the Cheese: Grasping Fractions and Developing Procedures

Douglas A. Lapp, Ph.D.
Central Michigan University
lapp1da@cmich.edu
@DougLappPhD



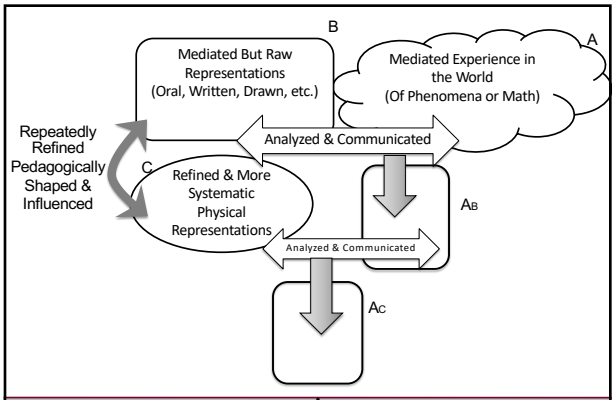
1

## Transition Among Representations

- Shifts between concept image and concept definition (Tall & Vinner, 1981).
  - Each have an affect on the other as symbolic meaning is negotiated.
- Process of building symbolic meaning is mediated by interactions among two worlds (the "real" world of *physical reality or ideas* and the world of the *symbols that represent these ideas*. (Kaput, Blanton, & Moreno, 2008; Lapp, Ermete, Brackett, & Powell, 2013; Lapp & Cyrus, 2000)

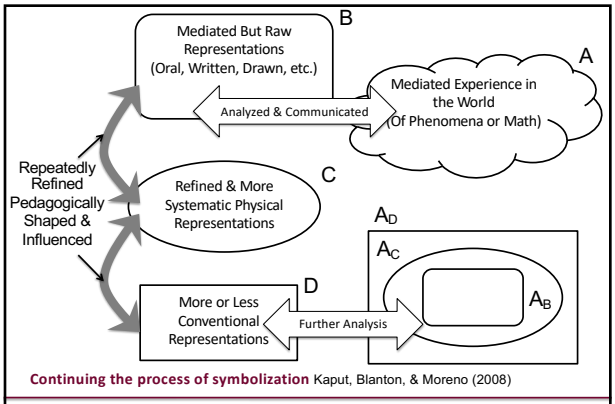


2






3




**Continuing the process of symbolization** Kaput, Blanton, & Moreno (2008)



4

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- Opening Discussion Questions
  - Since everyone got one piece of cheese, is this a fair way to give cheese?
  - If so, justify why do you think it is. If not, why do you think it isn't fair?

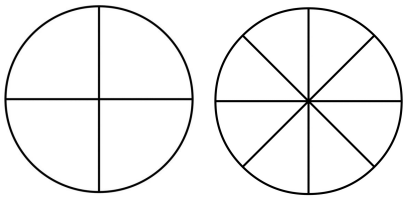


5

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- Describe how you decided to compare the cheese that everyone received.



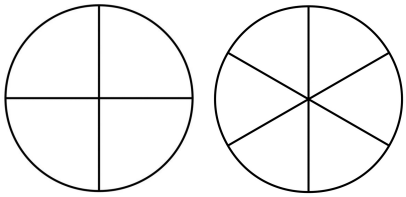


6

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- What if the slices of cheese were cut in this way and some received wedges from one circle and others received cheese from the other circle? How would you decide who got more?
- Could we make it fair by giving some people a single wedge from the first circle and others two wedges from the second?





7

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- If white is 1, purple is 4, and brown is 8, what is the smallest length train you can make of each color so that the trains end at the same length? What number does this length give?

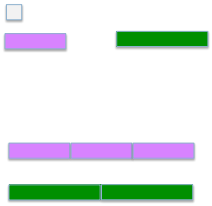


8

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- If white is 1, purple is 4, and dark green is 6, what is the smallest length train you can make of each color so that the trains end at the same length? What number does this length give?



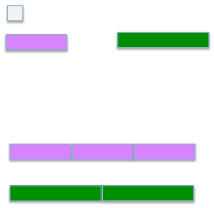


9

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- How many wedges would we have to cut both slices of cheese into so that we could compare how much each person received?
- Remember, we need to use the cut lines that are already in our cheese.



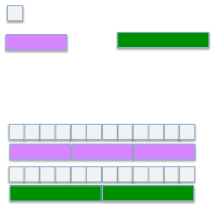


10

## Motivating Inquiry

Thought provoking questions requiring more than a simple response.

- Is this the smallest number of wedges we can use for both slices of cheese so that we can compare? How do you know?

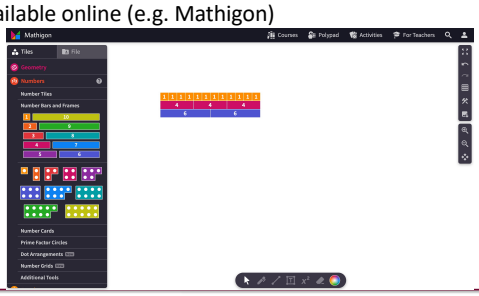




11

## Motivating Inquiry

Free virtual versions of these manipulatives are available online (e.g. Mathigon)






12

## Investigating Why: Linking Representations

What if we didn't have cheese or colored rods? How could we answer these questions with just the numbers of wedges there are for each slice of cheese?

- Remember when we took numbers and broke them into a product of primes. Let's explore how the prime factorization of numbers might help.

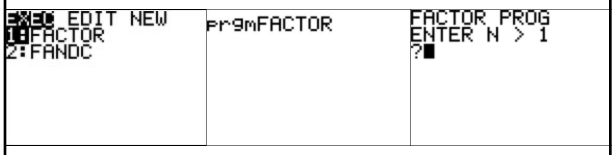


13

## Investigating Why: Linking Representations

First Case: 4 wedge slice and 8 wedge slice

Using the Factor program on your calculator, find the prime factorization of 4 and 8.



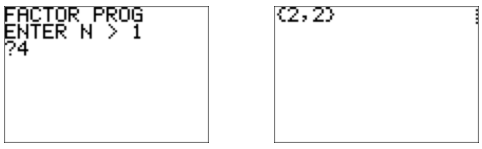



14

## Investigating Why: Linking Representations

First Case: 4 wedge slice and 8 wedge slice

Using the Factor program on your calculator, find the prime factorization of 4 and 8.




$$4 = 2 \cdot 2$$

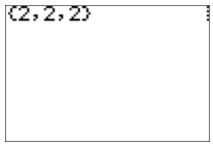


15

## Investigating Why: Linking Representations

First Case: 4 wedge slice and 8 wedge slice

Using the Factor program on your calculator, find the prime factorization of 4 and 8.


$$8 = 2 \cdot 2 \cdot 2$$



16

## Investigating Why: Linking Representations

Remember that in this case, you said that we need to have both slices of cheese cut in 8 wedges in order to compare them. In your groups, look at the prime factorization of 8 and see if you can "see" the number 4 inside it. Explain your thinking to your groupmates. Be ready to share your observations.

$$8 = 2 \cdot 2 \cdot 2$$

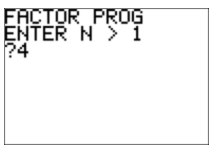


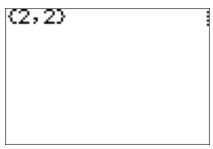
17

## Investigating Why: Linking Representations

Second Case: 4 wedge slice and 6 wedge slice

Using the Factor program on your calculator, find the prime factorization of 4 and 6.




$$4 = 2 \cdot 2$$

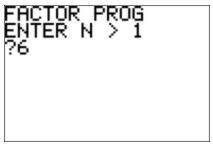


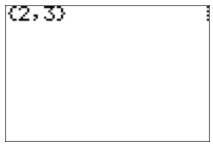
18

## Investigating Why: Linking Representations

Second Case: 4 wedge slice and 6 wedge slice

Using the Factor program on your calculator, find the prime factorization of 4 and 6.




$$6 = 2 \cdot 3$$



19

## Investigating Why: Linking Representations

Remember that in this case, you said that we need to have both slices of cheese cut in 12 wedges in order to compare them. In your groups, look at the prime factorization of 12 and see if you can "see" the number 4 inside it. Can you "see" the number 6 inside it? Explain your thinking to your groupmates. Be ready to share your observations.

$$12 = 2 \cdot 2 \cdot 3$$



20

## Testing Your Observations

Now that we have noticed some patterns, let's test these using the *least common multiple command*, lcm(a,b). Suppose we have cheese slices cut into 18 and 60 wedges. In your groups, find the lcm(18,60) and the factorizations of 18, 60, and the lcm(18,60)? Can you "see" both 18 and 60 in the factorization of the lcm(18,60)? Explain your thinking to your groupmates.

$$180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$



21

## Testing Your Observations

What if you just pushed the two factorizations for 18 and 60 together? Would this work? Can you "see" both 18 and 60 in the factorization of the product? Explain your thinking to your groupmates. Why is 180 better and how can you create it from the factors of 18 and 60 keeping the number as small as possible? Explain your thinking to your groupmates.

$$18 \cdot 60 = 1080 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$



22

## Sustaining Inquiry

- Students share control of the learning process.
  - Make decisions about what to do
  - Real World applications (not just word problems)
  - Debate the reasonableness of ideas
  - Require justification of claims/observations



23

## Rethinking the Teacher Role

- Teachers act as diagnosticians and not as the sole conveyers of knowledge.
  - Teacher as reflective listener
  - Teacher as orchestrator



24

## Physical Layout Matters

- Physical layout of the room
  - Teacher not in front
  - Students in tables for discussion
  - Student Grouping



25

## Student Thinking Is a Priority

Opportunity for reporting out observations/claims.

- First, within working groups (safe environment to share)
- Second, whole class (after group has discussed)



26